

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Discrete Mathematics

Subject Code : 4TE04DSM2

Branch: B. Tech (CE)

Semester : 4

Date : 15/04/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) Negation of $\exists x \forall y P(x, y)$ is
 (A) $\forall x \forall y \neg P(x, y)$ (B) $\forall x \exists y \neg P(x, y)$ (C) $\exists x \exists y \neg P(x, y)$
 (D) none of these
- b) The negation of “some students like football” is
 (A) Some students dislike football (B) Every student dislikes football
 (C) Every student likes football (D) none of these
- c) The number of binary relations of a set with n element is
 (A) n^2 (B) 2^n (C) 2^{n^2} (D) none of these
- d) The inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ is
 (A) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$
 (D) none of these
- e) In a group (G, \circ) if $(a \circ b)^{-1} = a^{-1} \circ b^{-1}$ then
 (A) G is finite (B) G is infinite (C) G is abelian (D) none of these
- f) The maximum number of zero element and unit element each in a poset is
 (A) 0 (B) 1 (C) 2 (D) none of these
- g) A self-complemented, distributive lattice is called
 (A) Boolean algebra (B) Modular lattice (C) Bounded lattice
 (D) Complete lattice
- h) In a lattice, if $a \leq b$ and $c \leq d$, then
 (A) $b \leq c$ (B) $a \leq d$ (C) $a \vee c \leq b \vee d$ (D) none of these
- i) If B is a Boolean Algebra, then which of the following is true



- (A) B is a finite but not complemented lattice.
 (B) B is a finite, complemented and distributive lattice.
 (C) B is a finite, distributive but not complemented lattice.
 (D) B is not distributive lattice.
- j) Let $*$ be a Boolean operation defined by $A * B = AB + \bar{A}\bar{B}$, then $A*A$ is:
 (A) A (B) B (C) 0 (D) 1
- k) A graph is a collection of
 (A) Row and columns (B) Vertices and edges (C) Equations
 (D) None of these
- l) A tree is _____.
 (A) always disconnected graph (B) always a connected graph
 (C) may be connected or disconnected (D) None of these
- m) Pigeonhole principle states that $A \rightarrow B$ and $|A| > |B|$ then:
 (A) f is not onto (B) f is not one-one (C) f is neither one-one nor onto
 (D) f may be one-one
- n) A debating team consists of 3 boys and 2 girls. Find the number of ways they can sit in a row?
 (A) 120 (B) 24 (C) 720 (D) 12

Attempt any four questions from Q-2 to Q-8

- Q-2 Attempt all questions (14)**
- a) Show that $\square r$ is a valid conclusion from the premises
 $p \Rightarrow \square q, r \Rightarrow p, q$ (a) with truth table (b) without truth table. (5)
- b) State and prove Lagrange's theorem on group. (5)
- c) Draw Hasse diagram for the poset $\langle S_{24}, \mathbf{D} \rangle$; where $a\mathbf{D}b$ means a divides b . (4)
- Q-3 Attempt all questions (14)**
- a) Find all subgroup of cyclic group of order 12 with generator a . Also find the order of each element of group G and find other generators of G . (5)
- b) Prove that $\langle \{1, 2, 3, 6\}, \text{GCD, LCM} \rangle$ is a sublattice of the lattice $\langle S_{30}, \text{GCD, LCM} \rangle$. (5)
- c) Find Join-irreducible elements and atoms for the lattice $\langle S_4 \times S_9, \mathbf{D} \rangle$. (4)
- Q-4 Attempt all questions (14)**
- a) Using definition of complement of an element find complement of each element of lattice $\langle S_{10}, \text{GCD, LCM, 1, 10} \rangle$ (5)
- b) Find all sub algebra of Boolean algebra $\langle S_{210}, *, \oplus, ', 0, 1 \rangle$. (5)
- c) Draw all non-isomorphic graph on 2 and 3 vertices. (4)
- Q-5 Attempt all questions (14)**
- a) State and prove Stone's representation theorem. (5)
- b) Draw the graph of tree represented by
 $(v_0 (v_1 (v_2) (v_3 (v_4) (v_5)))) (v_6 (v_7 (v_8)) (v_9) (v_{10}))$ (5)

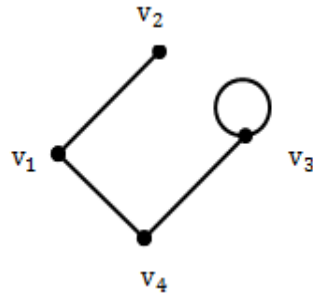


- c) Show that $3 + 33 + 333 + \dots + 33\dots\dots 3 = (10^{n+1} - 9n - 10) / 27$ (4)

By mathematical induction.

Q-6 **Attempt all questions** (14)

- a) Show that following graph is connected. (5)



- b) Show that in any room of people who have been doing handshaking there will always be at least two people who have shaken hands the same number of times. (5)

- c) Show that the following Boolean expression are equivalent. (4)

(i) $(x \oplus y) * (x' \oplus y)$, y

(ii) $x * (y \oplus (y' * (y \oplus y')))$, x

(iii) $(z' \oplus x) * ((x * y) \oplus z) * (z' \oplus y)$, $x * y$

Q-7 **Attempt all questions** (14)

- a) Prove that “Is a lattice isomorphism” is an equivalence relation on the set of all lattices. (5)

- b) From the following adjacency matrix, find the out degree and in degree of each node. Also verify your answer by drawing digraph and its adjacency matrix. (5)

$$\begin{matrix}
 & v_1 & v_2 & v_3 & v_4 \\
 v_1 & \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \\
 v_2 & \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix} \\
 v_3 & \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix} \\
 v_4 & \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}
 \end{matrix}$$

- c) Prove that the set of fourth roots of unity form a group under multiplication. (4)

Q-8 **Attempt all questions** (14)

- a) Determine all the proper subgroup of symmetric group (S_3, \cdot) . Which subgroup is normal? (5)

- b) Find all the minterms of a Boolean algebra with three variables x_1, x_2, x_3 . (5)

- c) Show that $(p \vee q) \wedge (\neg p \wedge \neg q)$ is a contradiction. (4)

